Switching Current Threshold in Planar Systems: An Analytical Approach in the Single Spin Approximation

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Several authors [e.g. 1-3] have derived expressions for the instability threshold of nanosize spinvalves submitted to a spin transfer torque. Instability thresholds deduced from essentially first order perturbation analysis are, however, conceptually distinct from switching thresholds. We consider in the following a particularly simple form of the magnetization dynamics master equation, namely

$$\frac{d\boldsymbol{m}}{d\tau} = -\left[\boldsymbol{m} \times \boldsymbol{h}_{Eff}\right] + \beta\left[\boldsymbol{m} \times (\boldsymbol{m} \times \boldsymbol{p})\right] + \alpha\left[\boldsymbol{m} \times \frac{d\boldsymbol{m}}{d\tau}\right] \qquad (1),$$

where all variables are dimensionless and both α, the damping parameter, and

$$\beta = \frac{\hbar}{2}\frac{1}{\mu_0 M_S^2}\frac{1}{d}\frac{J}{e} \qquad (2)$$

are small parameters. J is the current density, d the "free" layer thickness and $\boldsymbol{p}$ the magnetization direction within the "pinned" layer. Note that, in the present approach, the spin torque is symmetrical *vs* angle between $\boldsymbol{m}$ and $\boldsymbol{p}$. We also solely consider the case of a field applied along the easy magnetization axis, x. At rest, the stable magnetization directions are $m_x = \pm 1$. Following previous studies [4], we first fully characterize Hamiltonian trajectories ($\alpha = \beta = 0$) as a function of a single parameter, namely the energy (density), itself a function of the applied field. Under zero applied field, for instance, the Hamiltonian trajectories are defined by the intersection of the unit sphere with an hyperbolic cylinder (so-called "clam shell" trajectories). Trajectories increase in size with increasing energy. Still under zero applied field, trajectories are observed to pile-up against an homoclinic cycle defined by the intersection of the unit sphere with planes $m_z = \pm\sqrt{Q}\, m_x$ as the energy reaches half the in-plane anisotropy energy Q. z here is the normal to the "free" layer plane also referred to as the equatorial plane. Extension to the $h_x \neq 0$ case is straightforward: Hamiltonian trajectories keep growing until the field amplitude reaches the in-plane anisotropy field $h_x = Q$. For larger applied fields, Hamiltonian trajectories do split into two limit cycles, one above, one under the equatorial plane.

The magnetization motion equation (1) implies the following relation:

$$\left(\frac{d\boldsymbol{m}}{d\tau} \cdot \boldsymbol{h}_{Eff}\right) + \beta\left(\frac{d\boldsymbol{m}}{d\tau} \cdot (\boldsymbol{m} \times \boldsymbol{p})\right) - \alpha\left(\frac{d\boldsymbol{m}}{d\tau}\right)^2 = 0 \qquad (3),$$

stating that the work of the effective field augmented with that of the field $\boldsymbol{m} \times \boldsymbol{p}$ giving rise to the spin torque is balanced by dissipation.

Because, per definition, the work of the effective field is zero along any close trajectory Γ, it follows directly that

$$G = \oint_{\Gamma} \left[J \left(\frac{d\boldsymbol{m}}{d\tau} \cdot (\boldsymbol{m} \times \boldsymbol{p}) \right) - \alpha \left(\frac{d\boldsymbol{m}}{d\tau} \right)^2 \right] d\tau \equiv 0 \qquad (4).$$

Trajectories satisfying (4) are termed precessional states. Their very existence is a key prediction of spin transfer induced magnetization dynamics.

Melnikov's theory [5,6], on the other hand, when applicable, states that there exist physical trajectories close to the unperturbed (Hamiltonian) trajectories provided α and βJ remain small enough. It follows that the onset of precessional states is reached when the current density J becomes large enough in order to allow for the existence of an infinitesimal Hamiltonian trajectory satisfying (4) around one fixed point, say $m_x = 1$. The next threshold is reached when J becomes such that (4) is satisfied for the homoclinic cycle characteristic of a given applied field. The latter threshold defines the switching current.

For the simple energy landscape considered here, conversion of (4) to a line integral allows for an analytical solution for the critical current densities corresponding to both thresholds, namely

$$\frac{J^{Onset}}{\beta} = \alpha \left(\frac{1}{2} + Q + h_x \right)$$

$$\frac{J^{Switching}}{\beta} = r^2 \sqrt{1+Q}\, \alpha \left[\frac{h\theta_S + \sqrt{Q}\sqrt{1 - \dfrac{h_x^2}{Q^2}}}{h\sqrt{1 - \dfrac{h_x^2}{Q^2}} + r^2 \sqrt{Q}\,\theta_S} \right] \qquad (5),$$

where, $h = \dfrac{h_x}{Q}\sqrt{\dfrac{Q}{1+Q}}$; $r^2 = 1 - h^2$; $\theta_S = \cos^{-1}\left(\dfrac{h}{\sqrt{Q}\sqrt{1-h^2}} \right)$.

Based on an example, it will be shown that critical current densities deduced from Melnikov's theory are, within its validity range, undistinguishable from the corresponding critical current densities obtained by direct integration of Eqn.(1) [7]. Besides, Eqn.(5) embodies a weak dependence of $J^{Switching}$ on Q.

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